Dirac structures and Boundary Control Systems

Y. Le Gorrec
LAGEP UCBL
with B. Maschke, H. Zwart and J. Villegas

Introduction

Context

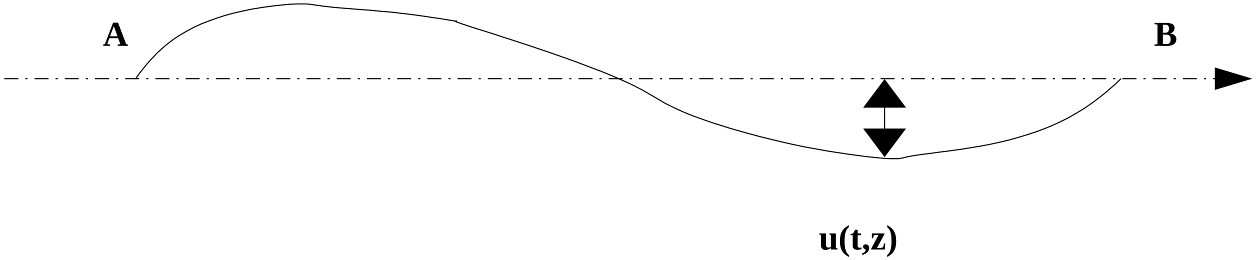
$\rightsquigarrow$ Collaboration LAGEP / University of Twente (The Netherland) within GEOPLEX () and Van Gogh projects.

$\rightsquigarrow$ *Network perspective* on physical systems' theory for modelling, simulation and control design. Following the same idea behind the **bond graph formalism** (in finite dimension) :

- interconnection of small set of atomic elements with particular energetic behavior (storing, dissipation or conversion)
- interaction with the environment by means of a port
- interdomain coupling using a network structure called Dirac structure

⇝ Systems described by PDE and skew symmetric operators : wave equations, beams (Bernuoilli, Timoshenko, ...). Existence and nature of solutions in function of the choice of the boundary conditions.

⇝ **Vibrating string**



The classical modelling is based on the wave equation : Newton's law + Hooke's law (restoring force proportional to the deformation)

$$\frac{\partial^2 u(z,t)}{\partial t^2} = \sigma^2 \frac{\partial^2 u(z,t)}{\partial z^2} \quad \text{where} \quad |\sigma| = \sqrt{\frac{T(z)}{\mu(z)}}$$

The structure of the model is not apparent. How to choose the boundary conditions ???

The energy and state variables

Let choose as state variables the energy variables:

- the strain $\epsilon = \frac{\partial u(z,t)}{\partial z}$
- the elastic momentum $p = \mu v(z,t)$

The total energy is given by : $H(\epsilon, p) = U(\epsilon) + K(p)$

- $U(\epsilon)$ is the elastic potential energy:

$$U(\epsilon) = \int_a^b \frac{1}{2} T(z) \left(\frac{\partial u(z,t)}{\partial z} \right)^2 = \int_a^b \frac{1}{2} T \epsilon(z,t)^2$$

  where $T(z)$ denotes the elastic modulus.

- $K(v)$ is the kinetic co-energy:

$$K(p) = \int_a^b \frac{1}{2} \mu(z) v\,(z,t)^2 = \int_a^b \frac{1}{2} \frac{1}{\mu(z)} p^2(z,t)$$

  where $\mu(z)$ denotes the string mass.

Interdomain coupling

The vector of fluxes is given by:

$$\beta = \begin{pmatrix} v(t,z) \\ \sigma \end{pmatrix}$$

where $v(z,t)$ is the velocity and $\sigma = T\epsilon$ the stress.
The vector of fluxes β may be expressed in term of the generating forces :

$$\beta = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\text{canonical inerdomain coupling}} \underbrace{\begin{pmatrix} \frac{\delta H}{\delta \epsilon} \\ \frac{\delta H}{\delta p} \end{pmatrix}}_{\text{generating forces}}$$

Hamiltonian formulation

From the conservation laws:

$$\frac{\partial}{\partial t}\begin{pmatrix}\epsilon\\ p\end{pmatrix}+\frac{\partial}{\partial z}\begin{pmatrix}v\\ \sigma\end{pmatrix}=0$$

Consequently

$$\frac{\partial}{\partial t}\begin{pmatrix}\epsilon\\ p\end{pmatrix}=-\frac{\partial}{\partial z}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\frac{\delta H}{\delta\epsilon}\\ \frac{\delta H}{\delta p}\end{pmatrix}$$

and

$$\frac{\partial}{\partial t}\begin{pmatrix}\epsilon\\ p\end{pmatrix}=\begin{pmatrix}0 & -\frac{\partial}{\partial z}\\ -\frac{\partial}{\partial z} & 0\end{pmatrix}\begin{pmatrix}\frac{\delta H}{\delta\epsilon}\\ \frac{\delta H}{\delta p}\end{pmatrix}$$

Skew symmetry

$$\underbrace{\frac{\partial}{\partial t}\begin{pmatrix}\epsilon\\ p\end{pmatrix}}_{f} = \underbrace{\begin{pmatrix}0 & -\frac{\partial}{\partial z}\\ -\frac{\partial}{\partial z} & 0\end{pmatrix}}_{\mathcal{J}=\text{matrix differential operator}} \underbrace{\begin{pmatrix}T(z) & 0\\ 0 & \frac{1}{\mu(z)}\end{pmatrix}\begin{pmatrix}\epsilon\\ p\end{pmatrix}}_{e=\text{driving force}}$$

Hamiltonian operator $\mathcal{J}$ is **skew-symmetric only for function with compact domain strictly** in Z :

$$\int_a^b \begin{pmatrix} e_1 & e_2\end{pmatrix}\mathcal{J}\begin{pmatrix}e_1'\\ e_2'\end{pmatrix} + \begin{pmatrix}e_1' & e_2'\end{pmatrix}\mathcal{J}\begin{pmatrix}e_1\\ e_2\end{pmatrix} = -\left[e_1e_2' + e_2e_1'\right]_a^b$$

Power balance

Power balance equation :

$$
\begin{aligned}
\frac{d}{dt}H &= \int_a^b \left(\frac{\delta \mathcal{H}}{\delta \alpha_1} \frac{\partial \alpha_1}{\partial t} + \frac{\delta \mathcal{H}}{\delta \alpha_2} \frac{\partial \alpha_2}{\partial t} \right) dz \\
&= -\int_a^b \left(\frac{\delta \mathcal{H}}{\delta \alpha_1} \frac{\partial}{\partial z} \frac{\delta \mathcal{H}}{\delta \alpha_2} + \frac{\delta \mathcal{H}}{\delta \alpha_2} \frac{\partial}{\partial z} \frac{\delta H}{\delta \alpha_1} \right) dz \\
&= -\left[\frac{\delta \mathcal{H}}{\delta \alpha_1} \frac{\delta \mathcal{H}}{\delta \alpha_2} \right]_a^b
\end{aligned}
$$

If driving forces are zero at the boundary, the total energy is conserved, else there is a **flow of power at the boundary**. Define two **port boundary variables** as follows :

$$
\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \begin{pmatrix} \frac{\delta H}{\delta v} \\ \frac{\delta H}{\delta u} \end{pmatrix} \Big|_{a,b}
$$

Port Hamiltonian Formulation

The linear space $\mathcal{D} \ni (f_1, f_2, e_1, e_2, f_\partial, e_\partial)$

- $\begin{pmatrix} f_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\partial}{\partial z} \\ -\frac{\partial}{\partial z} & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$
- $\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} |_{a,b}$

defines a **Dirac structure:**$\mathcal{D} = \mathcal{D}^\perp$ with respect to the symmetric pairing :

$$\int_a^b e_1 f_1 dz + \int_a^b e_2 f_2 dz + [f_\partial e_\partial]_a^b$$

Port Hamiltonian system

$$\left(\frac{\partial}{\partial t}\alpha, \frac{\delta H}{\delta \alpha}, f_\partial, e_\partial \right) \in \mathcal{D}$$

Port Hamiltonian Formulation

An other choice for the state variables : $\begin{pmatrix} u \\ v \end{pmatrix}$ the Hamiltonian system becomes

$$\frac{\partial x}{\partial t} = \begin{pmatrix} 0 & \frac{1}{\mu} \\ \frac{1}{\mu} & 0 \end{pmatrix} \begin{pmatrix} \frac{\delta H}{\delta v} \\ \frac{\delta H}{\delta u} \end{pmatrix}$$

with port variables at the boundary:

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \begin{pmatrix} v \\ T\frac{\partial u}{\partial z} \end{pmatrix} |_{a,b}$$

which have a non trivial relation with the variational derivatives:

$$\frac{\delta H}{\delta x} = \begin{pmatrix} -\frac{\partial}{\partial z}\left(T\frac{\partial u}{\partial z}\right) \\ \mu v \end{pmatrix}$$

Focus

Generalization of the Port Hamiltonian Systems to infinite dimensional systems

- Canonical formulation of Dirac structures for infinite dimensional systems
- Existence of solution using the semi group theory : the Boundary Control System formulation
- Stabilizing controllers based on the *shaping of the energy and geometric structure*

Dirac Structure and PHS defined on Hilbert Space

General formulation

The system is defined by :

$$f = \mathcal{J} e$$

- Let the space of flow variables, $\mathcal{F}$, and the space of effort variables,$\mathcal{E}$, be real Hilbert spaces.
- Define the space of bond variables as $\mathcal{B} = \mathcal{F} \times \mathcal{E}$ endowed by the natural inner product

$$\langle b^1, b^2 \rangle = \langle f^1, f^2 \rangle_{\mathcal{F}} + \langle e^1, e^2 \rangle_{\mathcal{E}}, \quad b^1 = \left(f^1, e^1\right), b^2 = \left(f^2, e^2\right) \in \mathcal{B}$$

In order to define a Dirac structure, let us moreover endow the bond space $\mathcal{B}$ with a *canonical symmetrical pairing*, i.e., a bilinear form defined as follows:

$$\langle b^1, b^2 \rangle_+ = \langle f^1, r_{\mathcal{E},\mathcal{F}} e^2 \rangle_{\mathcal{F}} + \langle e^1, r_{\mathcal{F},\mathcal{E}} f^2 \rangle_{\mathcal{E}}, b^1 = \left(f^1, e^1\right), b^2 = \left(f^2, e^2\right) \in \mathcal{B}. \tag{1}$$

Definition on Hilbert space

Denote by $\mathcal{D}^{\perp}$ the orthogonal subspace to $\mathcal{D}$ with respect to the symmetrical pairing:

$$\mathcal{D}^{\perp} = \left\{ b \in \mathcal{B} | \left\langle b, b' \right\rangle_{+} = 0 \text{ for all } b' \in \mathcal{D} \right\}. \qquad (2)$$

Definition : A Dirac structure $\mathcal{D}$ on the bond space $\mathcal{B} = \mathcal{F} \times \mathcal{E}$ is a subspace of $\mathcal{B}$ which is maximally isotropic with respect to the canonical symmetrical pairing, i.e.,

$$\mathcal{D}^{\perp} = \mathcal{D}. \qquad (3)$$

$$\begin{pmatrix} f \\ e \end{pmatrix} \in \mathcal{D} \Longleftrightarrow \textbf{Power conservation}$$

Port Hamiltonian System PHS

$\rightsquigarrow$ Definition based on Dirac structure and Hamiltonian function (total energy of the system).

Definition : Let $\mathcal{B} = \mathcal{E} \times \mathcal{F}$ be the bound space defined above and consider the Dirac structure $\mathcal{D}$ and the Hamilonian function $\mathcal{H}(x)$ with x the energy variables. Define the flow variables, $f \in \mathcal{F}$ as the time variation of the energy variables and the effort variables $e \in \mathcal{E}$ as the variational derivative of $\mathcal{H}(x)$. The system

$$(f, e) = \left(\frac{\partial x}{\partial t}, \frac{\delta \mathcal{H}}{\delta x} \right) \in \mathcal{D}$$

is a Port Hamiltonian system with total energy $\mathcal{H}(x)$

We start by introducing the differential operator $\mathcal{J}$

$$\mathcal{J}e = \sum_{i=0}^{N} P(i)\frac{d^i e}{dz^i}(z) \qquad z \in [a,b]\,,$$

where $e \in H^N((a,b);\mathbb{R}^n)$ and $P(i)$, $i = 0,\dots,N$, is a $n \times n$ real matrix with P_N non singular and $P_i = P_i^T(-1)^{i+1}$. Let define

$$Q = \begin{pmatrix} P_1 & P_2 & \cdots & P_N \\ -P_2 & -P_3 & \cdots & 0 \\ \vdots & \dots & \ddots & \vdots \\ (-1)^{N-1}P_N & 0 & \cdots & 0 \end{pmatrix}$$

Back to the **Vibrating string**

$$\underbrace{\frac{\partial}{\partial t}\begin{pmatrix} \epsilon \\ p \end{pmatrix}}_{f} = \underbrace{\begin{pmatrix} 0 & -\frac{\partial}{\partial z} \\ -\frac{\partial}{\partial z} & 0 \end{pmatrix}}_{P_1} \underbrace{\begin{pmatrix} T(z) & 0 \\ 0 & \frac{1}{\mu(z)} \end{pmatrix}\begin{pmatrix} \epsilon \\ p \end{pmatrix}}_{e} \,, Q = P_1$$

Canonical symmetrical pairing and port varia

$\rightsquigarrow$ We define the symmetrical pairing (not depending on $\mathcal{J}$) of and the port variables associated with $\mathcal{J}$.
Let $\mathcal{F} = \mathcal{E} = L^2((a,b);\mathbb{R}^n) \times \mathbb{R}^{nN}$ and define $\mathcal{B} = \mathcal{F} \times \mathcal{E}$ with the following canonical symmetrical pairing :

$$\begin{aligned}&\left\langle \left(f^1, f^1_\partial, e^1, e^1_\partial\right) \left(f^2, f^2_\partial, e^2, e^2_\partial\right)\right\rangle_+ \\ &\qquad = \langle e^1, f^2\rangle_{L^2} + \langle e^2, f^1\rangle_{L^2} - \langle e^1_\partial, f^2_\partial\rangle - \langle e^2_\partial, f^1_\partial\rangle,\end{aligned}$$

Definition : The port variables $(e_\partial, f\partial) \in \mathbb{R}^{nN}$ associated with $\mathcal{J}$ are defined by :

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = R_{\text{ext}} \begin{pmatrix} e(b) \\ \vdots \\ \frac{d^{N-1}e}{dz^{N-1}}(b) \\ e(a) \\ \vdots \\ \frac{d^{N-1}e}{dz^{N-1}}(a) \end{pmatrix}, \quad R_{\text{ext}} = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & -Q \\ I & I \end{pmatrix}$$

Introduction

Dirac Structure and PHS defined on Hilbert Space
- Dirac structure
- PHS
- Constructive formulation of DS

Boundary Control Systems (BCS)

Closed loop BCS, stabilty etc ...

Conclusion and further works

References

Theorem : The subspace $\mathcal{D}_\mathcal{J}$ of $\mathcal{B}$ defined as

$$\mathcal{D}_\mathcal{J} = \left\{ \begin{pmatrix} f \\ f_\partial \\ e \\ e_\partial \end{pmatrix} \;\middle|\; e \in H^N((a,b);\mathbb{R}^n),\ \mathcal{J}e = f,\ \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = R_{\text{ext}} \begin{pmatrix} e(b) \\ \vdots \\ \partial_z^{N-1} e(a) \end{pmatrix} \right\}$$

is a Dirac structure, that means that $\mathcal{D} = \mathcal{D}^\perp$.

Back to the **Vibrating string**

$$\underbrace{\frac{\partial}{\partial t}\begin{pmatrix} \epsilon \\ p \end{pmatrix}}_{f} = \underbrace{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}}_{P_1} \frac{\partial}{\partial z} \underbrace{\begin{pmatrix} T(z)\epsilon \\ \frac{1}{\mu(z)} p \end{pmatrix}}_{e}, Q = P_1$$

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1 & -P_1 \\ I & I \end{pmatrix} \begin{pmatrix} e(b) \\ e(a) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} T(a)\epsilon(a) - T(b)\epsilon(b) \\ \frac{p(a)}{\mu(a)} - \frac{p(b)}{\mu(b)} \\ T(a)\epsilon(a) + T(b)\epsilon(b) \\ \frac{p(a)}{\mu(a)} + \frac{p(b)}{\mu(b)} \end{pmatrix}$$

Boundary Control Systems

Introduction

Dirac Structure and PHS defined on Hilbert Space

Boundary Control Systems (BCS)
- Input parametrization
- Output and energy balance

Closed loop BCS, stabilty etc ...

Conclusion and further works

References

Theorem : Let W be a $nN \times 2nN$ full rank matrix. The system

$$\dot{x}(t) = \mathcal{J} e(t)$$

$$u(t) = \mathcal{B} x(t) = W \begin{pmatrix} f_\partial(t) \\ e_\partial(t) \end{pmatrix}$$

is a boundary control system, where $A_W = (\mathcal{J}\mathcal{L})_{ker\mathcal{B}}$ is the generator of a contraction semigroup on $L_2\left((a,b), \mathbb{R}^n\right)$ if and only if

$$W \Sigma W^T \geq 0 \text{ where } \Sigma = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$W \Sigma W^T \geq 0 \;\Leftrightarrow\; W = S\,(I+V, I-V)$$

with S invertible and $VV^T \leq I$

Let define the linear mapping $\mathcal{C} : H^N((a,b),\mathbb{R}^n) \rightarrow \mathbb{R}^{nN}$ as

$$\mathcal{C}x(t) := S_2 \begin{pmatrix} I - V^T & -I - V^T \end{pmatrix} \begin{pmatrix} f_\partial(t) \\ e_\partial(t) \end{pmatrix}$$

and the output as $y(t) = \mathcal{C}x(t)$, then for $u \in C^2((0,\infty);\mathbb{R}^{nN})$ and $x(0) - Bu(0) \in D(J_W)$ the following balance equation is satisfied:

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2 = \begin{pmatrix} u^T(t) & y^T(t) \end{pmatrix} P_W \begin{pmatrix} u(t) \\ y(t) \end{pmatrix}.$$

where $P_W = \begin{pmatrix} \frac{1}{4}S^{-T}(P_1^2 - P_1 V V^T P_1)S^{-1} & -2S^{-T}P_1 V P_2 S^T \\ -2SP_2V^TP_1S^{-1} & 4S(-P_2^2 + P_2V^TVP_2)S^T \end{pmatrix},$

$$P_1 = (I + VV^T)^{-1}, \qquad P_2 = (I + V^TV)^{-1}.$$

Particular cases

$$V = 0 \begin{cases} \dot{x}(t) & = \mathcal{J} x(t), \\ u(t) & = \frac{1}{2}\left(f_\partial(t) + e_\partial(t)\right) \\ y(t) & = \frac{1}{2}\left(f_\partial(t) - e_\partial(t)\right) \end{cases} \Longrightarrow$$

boundary control system, wi

the associated semigroup

a contraction

$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2 = \|u(t)\|^2 - \|y($

$$V = I \begin{cases} \dot{x}(t) & = \mathcal{J} x(t) \\ u(t) & = f_\partial(t) \\ y(t) & = -e_\partial(t) \end{cases} \Longrightarrow$$

boundary control system, wi

the associated semigroup

unitary

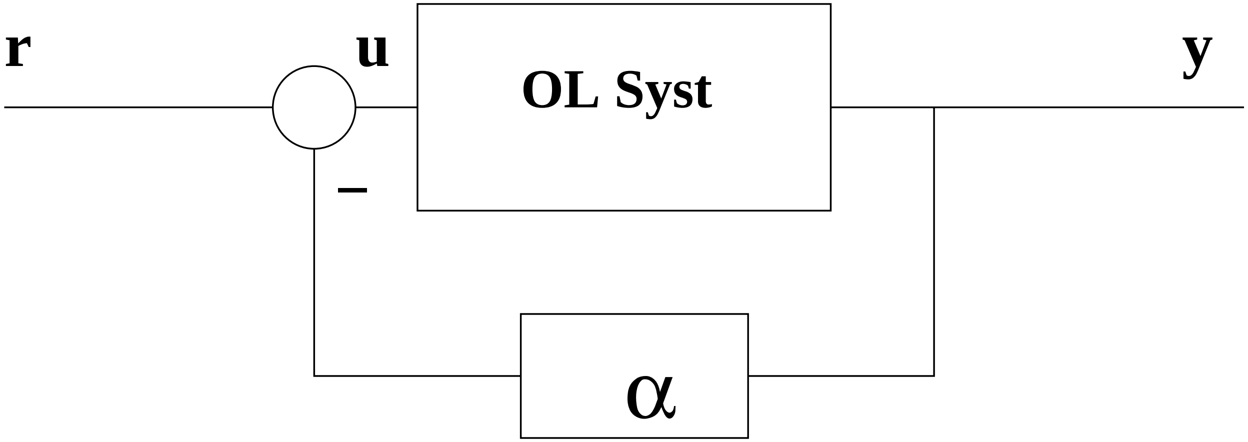
$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2 = u(t)^T y(t)$

Vibrating string :

$$V = 0 \quad \Rightarrow \quad u = \frac{1}{\sqrt{2}}\begin{pmatrix} T(a)\epsilon(a) \\ \frac{p(a)}{\mu(a)} \end{pmatrix} \text{ and } y = \frac{1}{\sqrt{2}}\begin{pmatrix} -T(b)\epsilon(b) \\ -\frac{p(b)}{\mu(b)} \end{pmatrix}$$

Closed loop BCS, stabilty etc ...

Static feedback for IPS



Open loop system

$$\dot{x} = \mathcal{J}x$$

$$u = W_{imp}\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

$$y = C_{imp}\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

Closed loop system

$$\dot{x} = \mathcal{J}x$$

$$r = (W_{imp} + \alpha C_{imp})\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

$$y = C_{imp}\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

Introduction

Dirac Structure and PHS defined on Hilbert Space

Boundary Control Systems (BCS)

Closed loop BCS, stabilty etc ...

- Static feedback for IPS
- Stability of IPS
- Generalization

Conclusion and further works

References

Lemma : The closed loop system described by

$$\dot{x} = \mathcal{J}_{\mathcal{L}} x$$

$$r = (W_{imp} + \alpha C_{imp}) \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

$$y = C_{imp} \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

is a boundary control system. Furthermore, the operator $A_s = \mathcal{J}_{\mathcal{L}|D(A_s)}$ generates a contraction semigroup on $X = L_2\left((a,b);\mathbb{R}^n\right)$ where

$$D(A_s) = \left\{ x \in D(\mathcal{J}) | \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} \in ker\tilde{W} \right\}$$

and $\tilde{W} = (W_{imp} + \alpha C_{imp})$ is a full rank $nN \times 2nN$ matrix

Theorem : $(\lambda - A_s)^{-1} : X \to X$ is a compact operator for $\lambda > 0$. Then the system described by $(VV^T = 0)$:

$$\dot{x} = \mathcal{J} x$$

$$r = (W_{imp} + \alpha C_{imp}) \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

$$y = C_{imp} \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

with $r = 0$ is globally asymptotically stable. For any $x(0) \in X$ the unique (classical or weak) solution $x(t) = T(t)x(0)$ of the closed loop system asymptotically approaches to zero, i.e.

$$lim_\infty \|x(t)\|_X = 0$$

Theorem : If $W\Sigma W^T > 0$ the system described by :

$$\dot{x} = \mathcal{J}x$$
$$r = W\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$
$$y = C\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix}$$

with $r = 0$ is **globally asymptotically stable**. For any $x(0) \in X$ the unique (classical or weak) solution $x(t) = T(t)x(0)$ of the closed loop system asymptotically approaches to zero, i.e.

$$lim_\infty \|x(t)\|_X = 0$$

Conclusion and further works

Conclusion and further works

⇝ Port hamiltonian formulation of Distributed parameter systems arises from two adjoint relations :

- conservation law
- generating forces being variational derivative of potential

and use of Stokes Theorem.

⇝ Underline the structural properties

⇝ Asymptotic stability can be checked by looking a condition on a matrix.

⇝ Links with well posed, conservative, boundary control systems.

⇝ Generalization to other operators using feedback ideas.

Future work :

⇝ Exponential stability ...

⇝ Controller design.

References

[1] Y. Le Gorrec, H. Zwart and B. Maschke, Dirac structures and Boundary Control Systems associated with Skew-Symmetric Differential Operators *SIAM Journal on Control and Optimization*, Vol: 44 Issue 5, pages 1864-1892, 2005.

[2] Y. Le Gorrec, H.Zwart and B. Maschke, A semigroup approach to Port Hamiltonian systems associated with linear skew-symmetric operators *In proc. of MTNS*, Leuven, 4-11 Jully 2004.

[3] J.A. Villegas, Y. Le Gorrec and H.Zwart, Boundary control systems and the system node *In proc. of IFAC World Congress*, Pragha, 25 Jully 2005.

[4] J. Villegas, H. Zwart, Y. Le Gorrec, B. Maschke, and A. J. van de Schaft, Stability and Stabilization of a Class of Boundary Control Systems *In proc. of the 45th IEEE CDC-ECC conference, Seville, Spain* , 2005.

[5] H. Zwart, Y. Le Gorrec, B. Maschke, J. Andres Villegas, Well-Posedness and regularity for a class of hyperbolic control systems *In proc. of the 17th International Symposium on Mathematical Theory of Networks and Systems* July 24-28, 2006, in Kyoto, Japan.

[6] J. Villegas, Y. Le Gorrec, H. Zwart, B. Maschke, Boundary control for a class of dissipative differential operators including diffusion systems *In proc. of the 17th International Symposium on Mathematical Theory of Networks and Systems* July 24-28, 2006, in Kyoto, Japan.