A control-oriented model of the flux diffusion in Tokamak plasma

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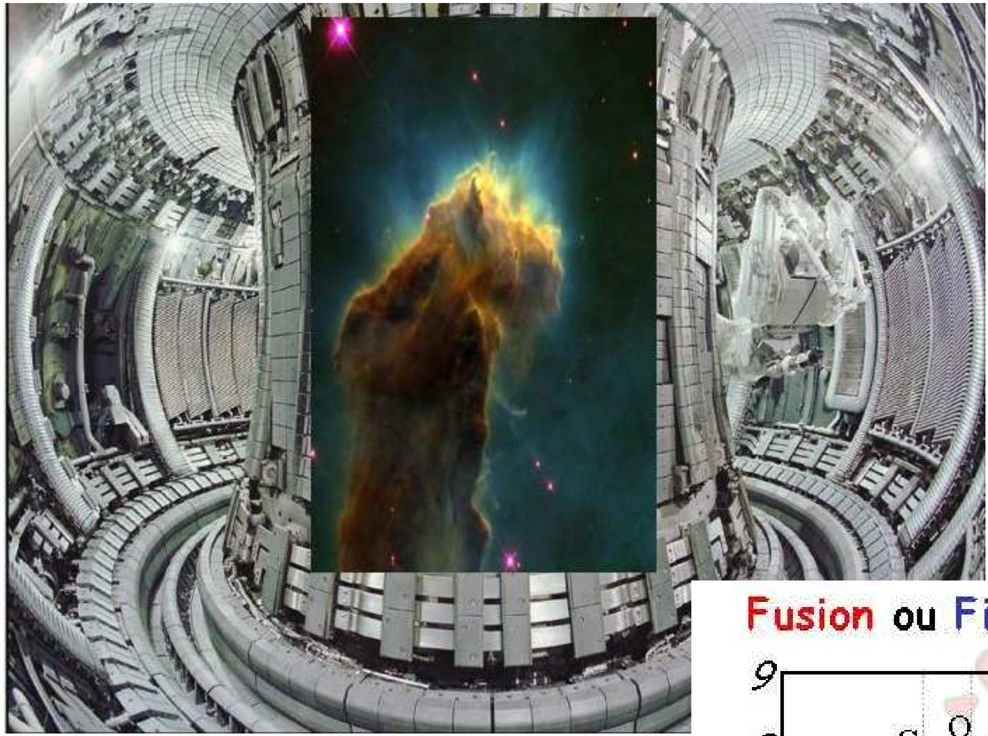
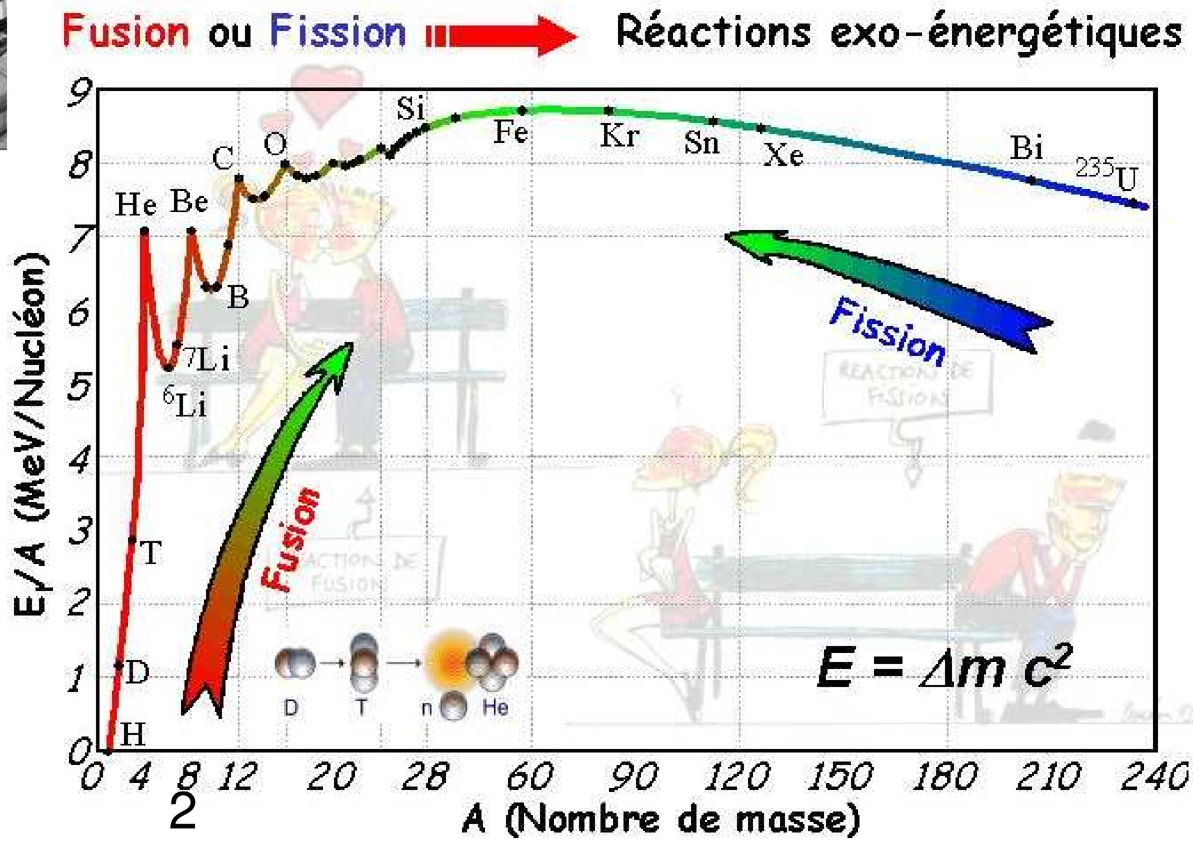
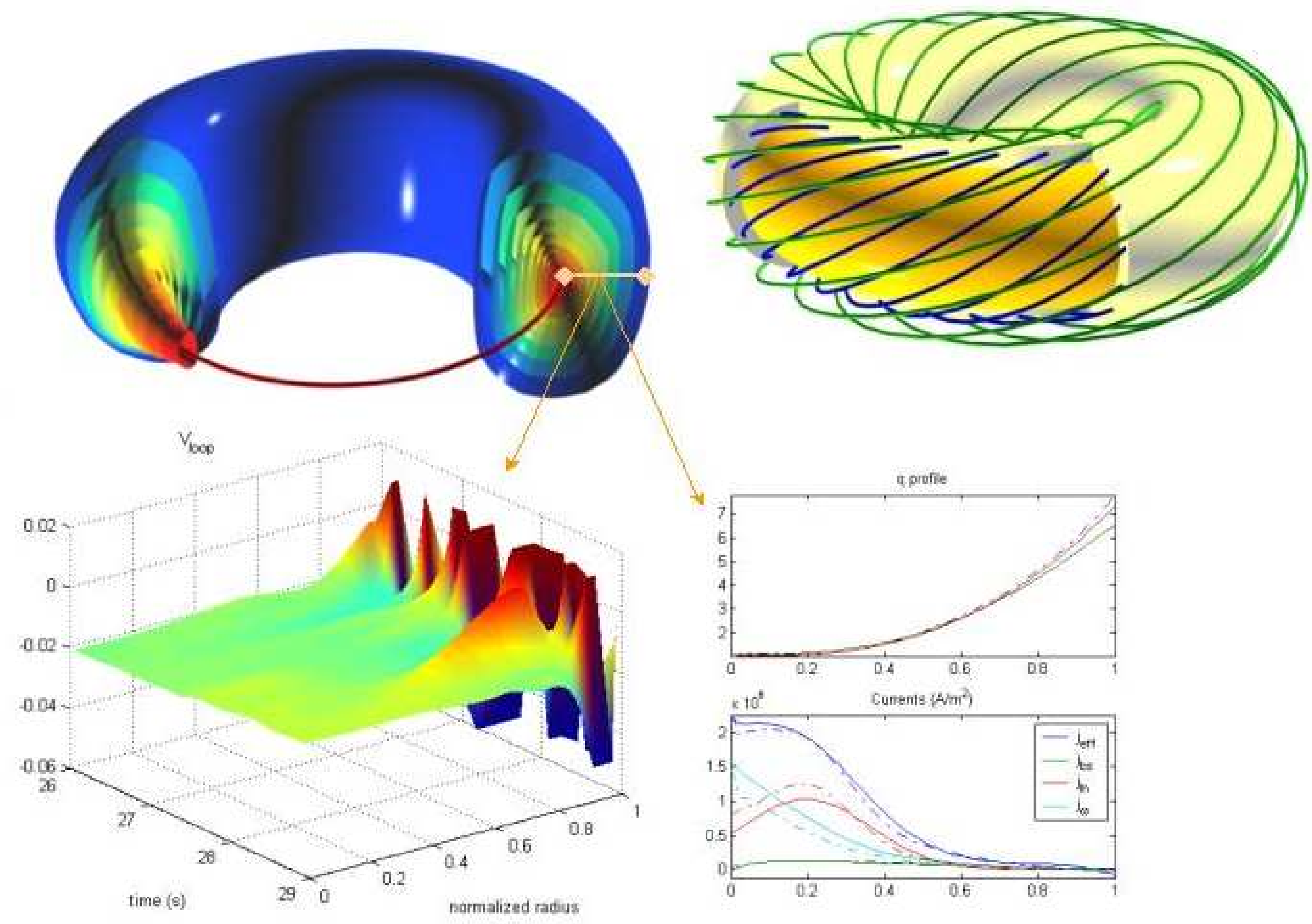
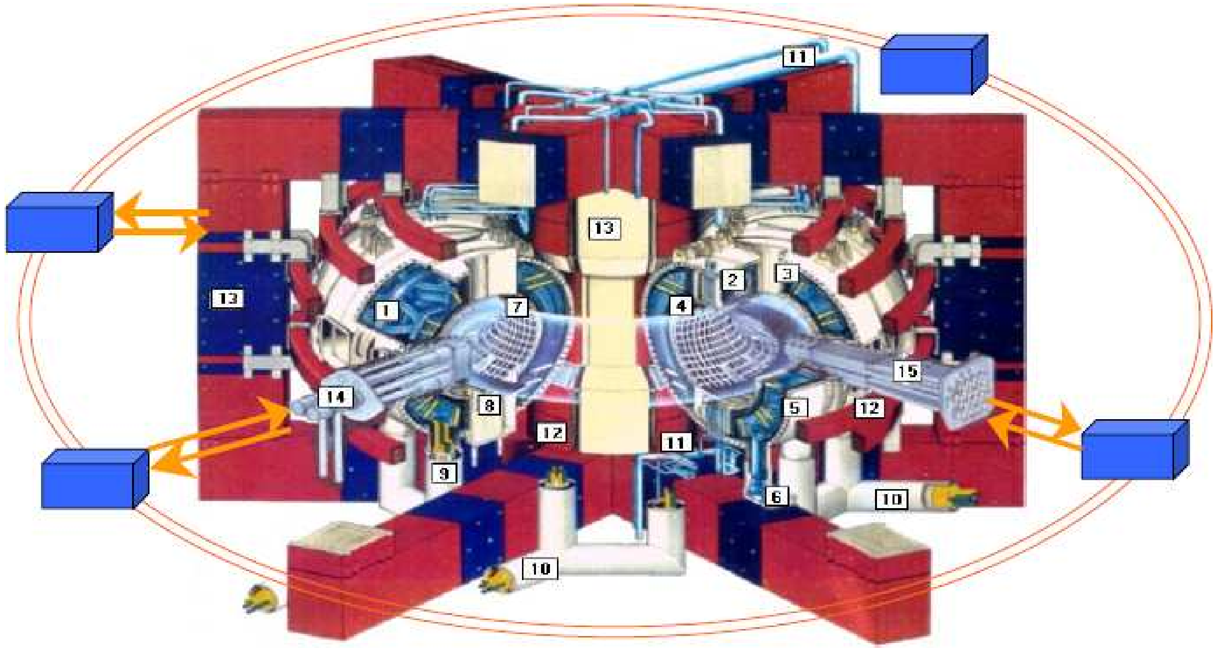



Fusion ou Fission
Réactions exo-énergétiques
E/A (MeV/Nucléon)
A (Nombre de masse)
0 4 8 12 20 28 60 90 120 150 180 210 240
0 1 2 3 4 5 6 7 8 9
H
D
T
^{6}Li
^{7}Li
He
Be
B
C
O
Si
Fe
Kr
Sn
Xe
Bi
^{235}U
Fusion
Fission
D T n He
$E = \Delta m c^2$



Vloop
0.02
0
-0.02
-0.04
-0.06
26
27
28
29
time (s)
0
0.2
0.4
0.6
0.8
1
normalized radius
q profile
Currents (A/m²)



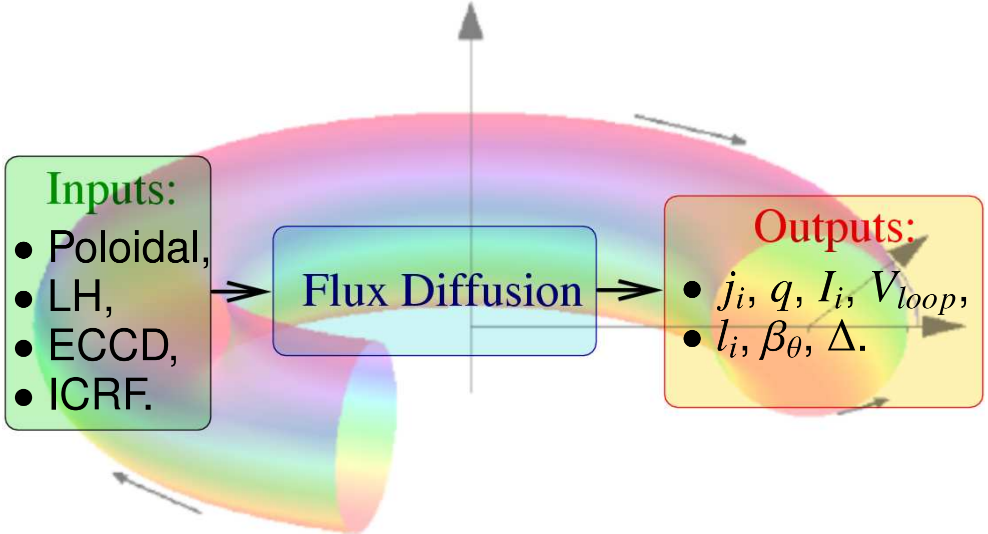
- Advanced plasma confinement schemes ⇒ current density profile control

⇒ representative RT model of plasma physics for control synthesis

- Focus on the magnetic flux dynamics,

- Include bootstrap effect, poloidal, LH, ECCD, (FCI).

II. Model dynamics



Hypothesis:

- cylindrical coordinates,
- neglect diamagnetic effect.

⇒ System dynamics [Blum'89, Brégeon & al'98]:

$$\frac{\partial \psi}{\partial t}(x,t) = \eta_{/\!/}(t)\left[\frac{1}{\mu_0 a^2}\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{\mu_0 a^2 x}\frac{\partial \psi}{\partial x} + R_0 j_{ni}(x,t)\right]$$

$$j_\phi(x,t) = -\frac{1}{\mu_0 R_0 a^2 x}\frac{\partial}{\partial x}\left[x\frac{\partial \psi}{\partial x}\right]$$

with $\psi'(0,t) = 0$, $\psi'(1,t) = f(I_p)$ or $\dot{\psi}(1,t) = f(V_{loop})$ and IC.

Resistivity, temperature and density profiles

- Resistivity [Hirshman'77]:

$$\eta_{/\!/}(t) = f(T_e, n_e, \bar{Z}) \propto T_e^{-3/2}(n_e, \bar{Z})$$

- Temperature: direct measurement or [ITER'99]

$$\begin{aligned}
\tau_{th} &= C I_p^{\alpha_I} B_{\phi_0}^{\alpha_B} P^{\alpha_P} \bar{n}_e^{\alpha_n} M^{\alpha_M} R_0^{\alpha_R} \epsilon^{\alpha_\epsilon} \kappa^{\alpha_\kappa} \\
\dot{W} &= P + \frac{W}{\tau_{th}} \\
< T_e > &= \frac{W}{6(1+\alpha_{Ti})\pi^2 e < n_e > \epsilon^2 \kappa R_0^3} \Rightarrow T_e(x,t) = T_0(1-x^2)^{\gamma_T}
\end{aligned}$$

- Density (average or interferometer):

$$n_{e0} = \frac{\gamma_n + 1}{\gamma_n} \bar{n}_e \quad and \quad n_e(x) = n_{e0}(1 - x^{\gamma_n})$$

- Bootstrap effect:
  - self induced current,
  - nonlinear component [Hirshman'88]:

$$
\begin{aligned}
j_{bs}(x,t) &= \frac{eR_0}{\partial\psi/\partial x}\left\{(A_1 - A_2)n_e\frac{\partial T_e}{\partial x} + A_1 T_e\frac{\partial n_e}{\partial x} + A_1(1-\alpha_i)n_i\frac{\partial T_i}{\partial x} + A_1 T_i\frac{\partial n_i}{\partial x}\right\} \\
&= \frac{eR_0}{\partial\psi/\partial x}\left\{\left(A_1\frac{\bar{Z} + \alpha_{T_i}(1-\alpha_i)}{\bar{Z}} - A_2\right)n_e\frac{\partial T_e}{\partial x} + A_1\frac{\bar{Z} + \alpha_{T_i}}{\bar{Z}}T_e\frac{\partial n_e}{\partial x}\right\}
\end{aligned}
$$

with $n_e = \bar{Z}n_i$ and $T_i = \alpha_{Ti}T_e$.

III. Model inputs

- BC input from the poloidal coil:

$$
\begin{aligned}
V_c &= R_c I_c + L_c \dot{I}_c + M \dot{I}_p \\
V_{loop} &= M \dot{I}_c - \frac{1}{I_p} \frac{\partial}{\partial t} \left[\frac{L_p I_p^2}{2} \right] = \dot{\psi}_a
\end{aligned}
$$

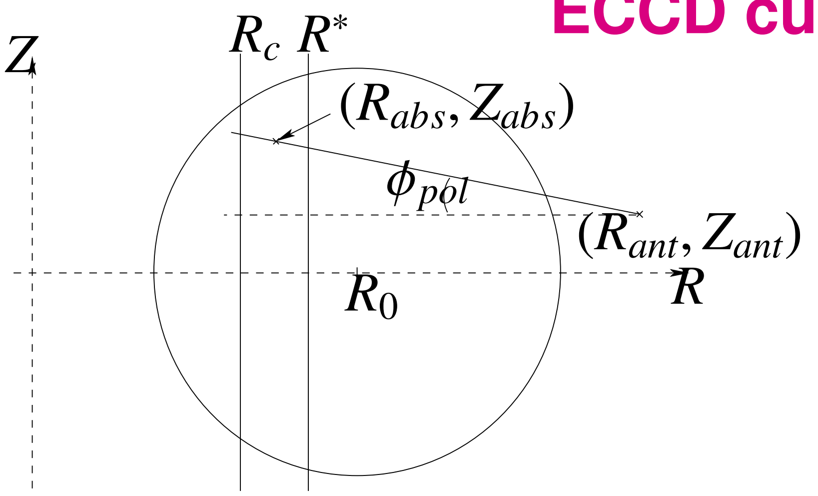
  Local control loop: set V_c according to $I_{p,ref}$ or $V_{loop,ref}$.

- Distributed current deposit from the antennas (LH, ECCD)
  ⇒ Engineering approach with gaussian approximation:

$$
j_{ni}(x,t) = \vartheta(t) e^{-(\mu(t)-x)^2/2\sigma(t)}
$$

- Temperature profile modification (ICRF, LH).

ECCD current deposit



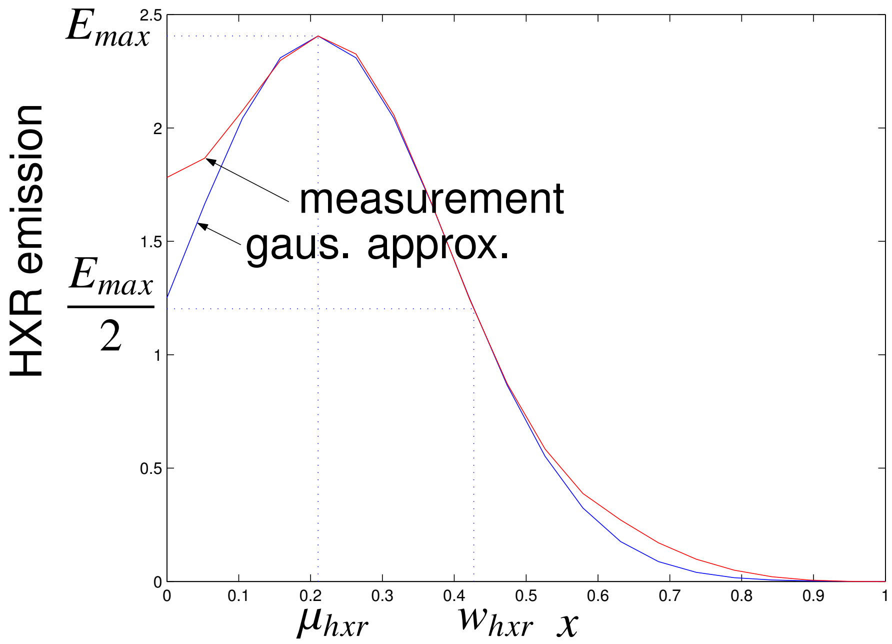
- Current deposit between R^* and R_c with [Giruzzi'87, Giruzzi & al.'91]:

$$R_c = 0.2044\, n_h \frac{I_{ind}}{f}, \quad R^*_m = \frac{R_{ant,m}}{\sqrt{2}|\sin\phi_{tor,m}|}\left[1-\sqrt{1-\frac{4R_c^2}{R^2_{ant,m}}\sin^2\phi_{tor,m}}\right]^{1/2},$$

$$\gamma_{cd,m} = \frac{\Gamma_1 T_e(x_{abs,m})}{T_e(x_{abs,m})+10^5}\left\{1-\Gamma_2\left[\frac{\rho_m(R_{abs,m},\phi_{pol,m})+R_{abs,m}-R_0}{R_{abs,m}}\right]^{\Gamma_3}\right\},$$

$$\vartheta_{cd,m} = \frac{\gamma_{cd,m}P_{cd,m}}{R_o\bar{n}_e}\left[2\pi a^2\int_0^1 xe^{-(\mu_{cd,m}-x)^2/2\sigma_{cd,m}}dx\right]^{-1} sign(\phi_{pol,m}).$$

LH current deposit



- Deposit shape:
  - directly from HXR measurement,
  - scaling laws:

$$\hat{w}_{hxr} = \alpha_0 B_{\phi_0}^{\alpha_1} I_p^{\alpha_2} \bar{n}^{\alpha_3} P_{lh}{}^{\alpha_4} N_{//}{}^{\alpha_5}$$

$$\hat{\mu}_{hxr} = \beta_0 B_{\phi_0}^{\beta_1} I_p^{\beta_2} \bar{n}^{\beta_3} P_{lh}{}^{\beta_4} N_{//}{}^{\beta_5}$$

- Current drive efficiency [Goniche & al.'05]:

$$\eta_{lh}(t) = 3.39 D_n^{0.26} \tau_{th}^{0.46} \bar{Z}^{-0.13},$$

with $D_n(t) \approx 2.03 - 0.63 N_{//}$

- Deposit magnitude:

$$I_{lh}(t) = \eta_{lh} \frac{P_{lh}}{\bar{n}_e R_0}$$

$$\vartheta_{lh}(t) = \frac{I_{lh}}{2\pi a^2 \int_0^1 x e^{-(\mu_{hxr}-x)^2/2\sigma_{lh}} dx}$$

IV. Discretization/Computation issues

- Main dynamics
  - variable step spatial discretization:

$$\dot{\psi}(x_i,t)_{ex} = \eta_{/\!/\, i,j}\left(e_1\psi_{i+1,j} - e_2\psi_{i,j} + e_3\psi_{i-1,j} + R_0 j_{ni\,i,j}\right)$$

  - Implicit-explicit temporal discretization:

$$\left(\frac{\psi_{i,j+1} - \psi_{i,j}}{\delta t}\right) = h\,\dot{\psi}(x_i,t)_{ex} + (1-h)\,\dot{\psi}(x_i,t)_{im}$$

  - final computation:

$$\begin{aligned} A_{i,i+1,j}\,\psi_{i+1,j} &+ A_{i,i,j}\,\psi_{i,j} + A_{i,i-1,j}\,\psi_{i-1,j} \\ -B_{i,i+1,j}\,\psi_{i+1,j+1} &- B_{i,i,j}\,\psi_{i,j+1} - B_{i,i-1,j}\,\psi_{i-1,j+1} + S_{i,j} = 0 \\ \Leftrightarrow \psi_{j+1} &= B_j^{-1}A_j\,\psi_j + B_j^{-1}S_j \end{aligned}$$

- Boundary conditions
  - central point not well defined:

$$\frac{\psi'(x_1,t)}{x_1} = \frac{0}{0} \Rightarrow \frac{1}{x}\frac{\partial}{\partial x}\left[x\frac{\partial\psi}{\partial x}\right] \approx \frac{1}{\delta x_2/4}\frac{\frac{\delta x_2}{2}\psi'_{1/2}}{\frac{\delta x_2}{2}} = \frac{4}{\delta x_2}\psi'_{1/2} \approx \frac{4}{\delta x_2^2}(\psi_2 - \psi_1)$$

  - border condition with I_p: set $\psi'_a(t)$ such that

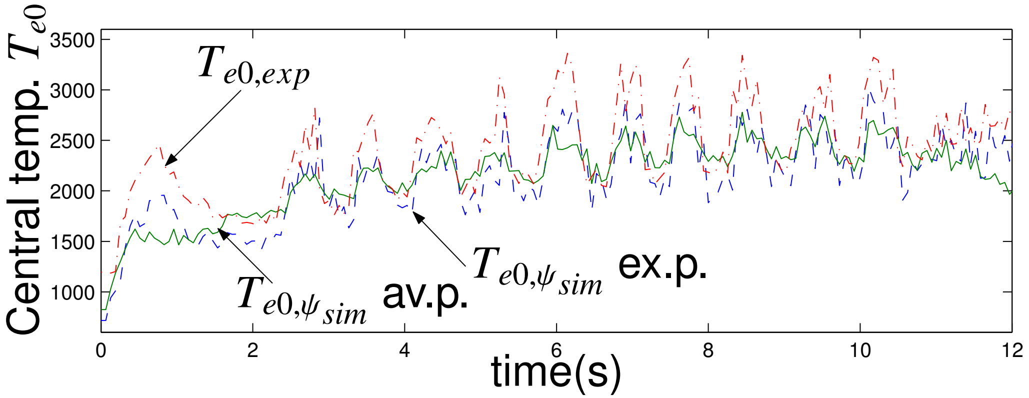
$$I_p - 2\pi a^2 \int_0^x x j_\phi(x,t) dx \quad \propto e^{-kt}$$

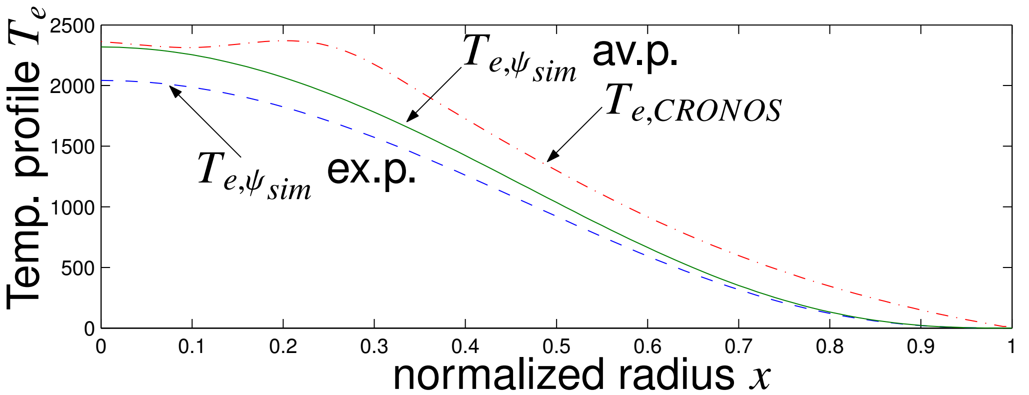
(computation error compensation)

V. Experimental/simulation results

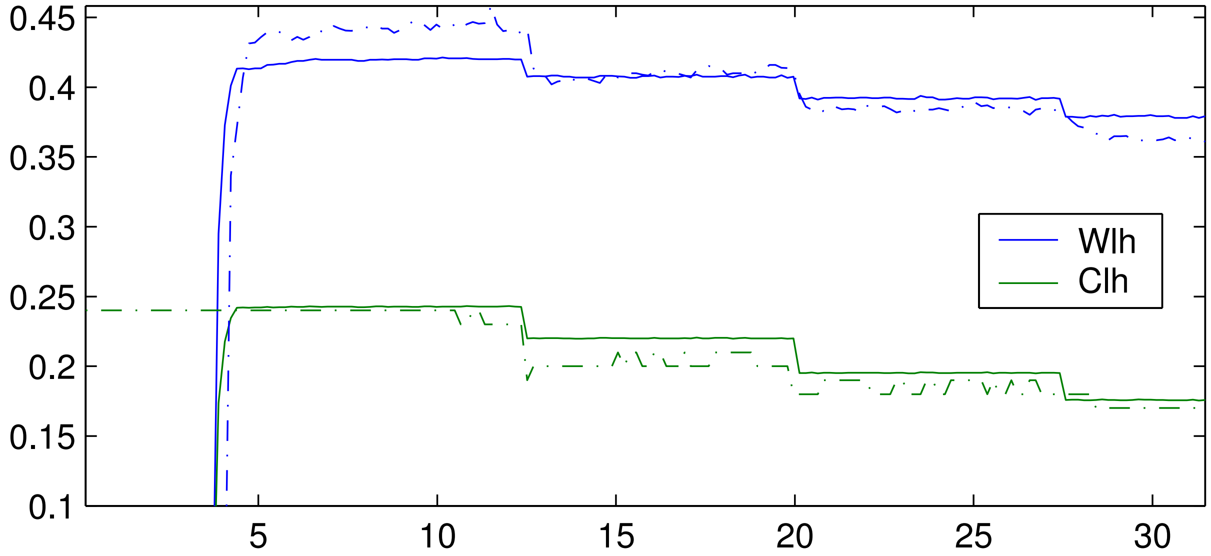
- shot TS 33632: $T(x,t)$ and $n(x,t)$ modulated with IRCF, and $I_p = 1.0\,MA$, $B_{\phi_0} = 3.19$, $n_{e0} = 4.5 - 3.0 \times 10^{-19}\,m^{-3}$.

- Temperature profile:

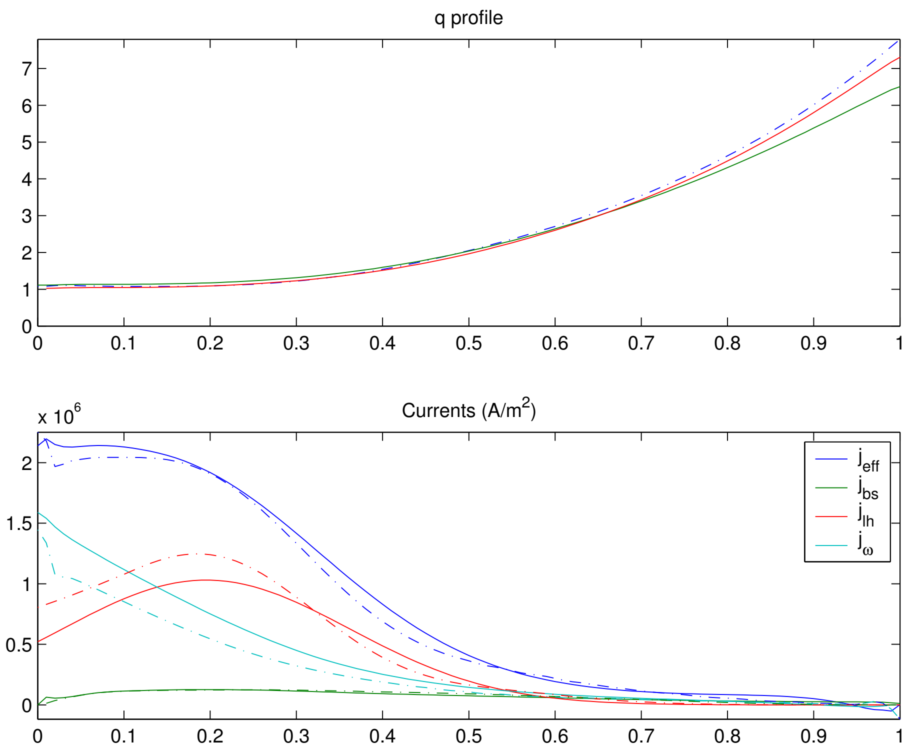




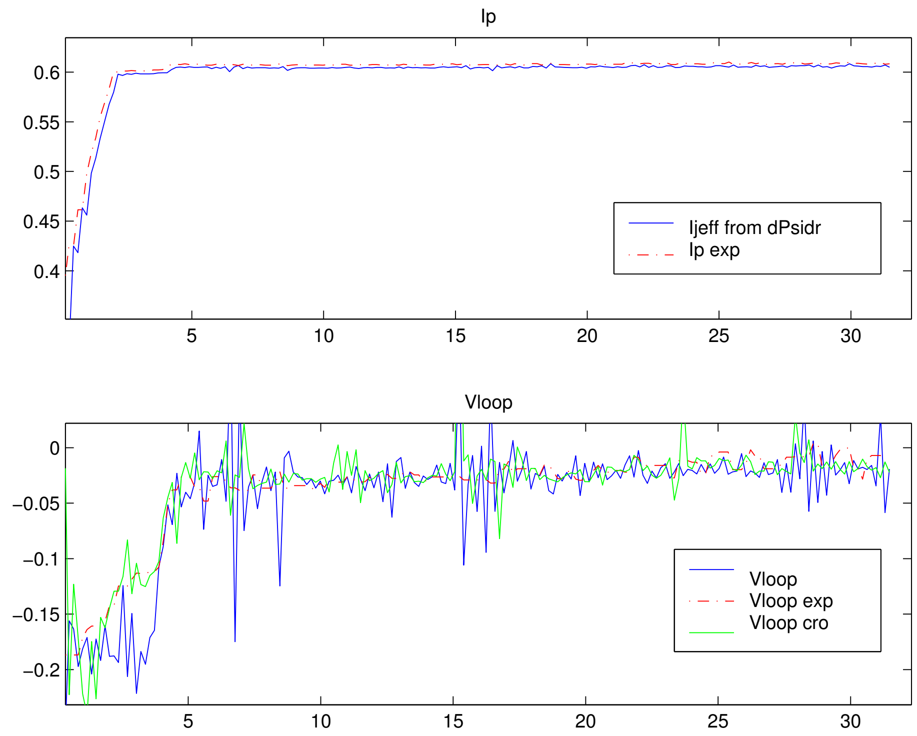
- shot TS 35109: variations in $N_{/\!/}$, constant I_p (0.6 MA) and power input (1.8 MW).

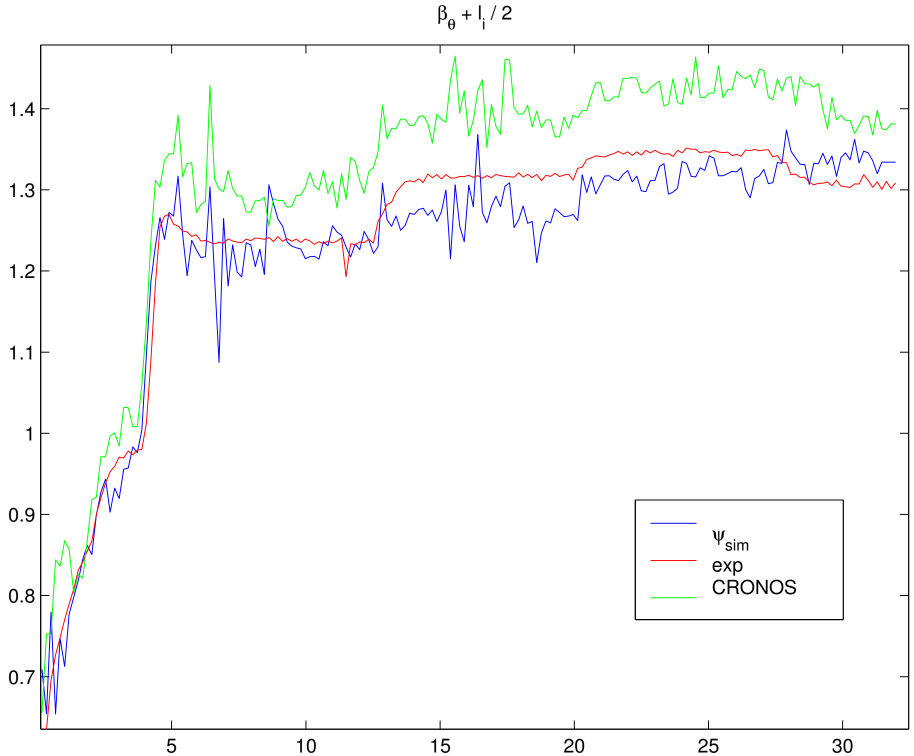


• Comparison with CRONOS: take $T_{e,i}$, $n_{e,i}$, η_{lh} as inputs.



• Plasma current and induced voltage: $T_{e,i}$, $n_{e,i}$ as inputs




$\beta_\theta + l_i / 2$
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
5
10
15
20
25
30
ψ_{sim}
exp
CRONOS

Conclusion

- Efficient model to represent the main control-relevant profiles,
- Fast computation ($\approx 1.4s$ for shot TS 35109 with 101 points) which allows for RT use,
- Variable step spatial discretization $\Rightarrow$ reduce the number of points (i.e. 20),
- Still needs to be validated for LH (T_e profiles) and FCE.

Prospects

1. Short term: "numerical" method: set the (stochastic) gradient descent with the model as an estimator:
   - $T_{e,i}$, $n_{e,i}$ as inputs, control $q(x_i, t)$, $i = 1..m$,
   - use ψ_{sim} to compute $(q_{ref} - q_m)$ and $S = \dfrac{\partial q}{\partial u_i}$ over a finite number of steps → optimal nonlinear control,
   - applied to infinite dimensional systems with stochastic inputs (TdS) → robustness,
   - associate with CUMSUM algorithm (threshold, i.e. on T_e + KF) to trigger u_i computation → adaptive/NLMP control,

   ⇒ reference performances.

2. Backstepping with distributed inputs (?)

3. Differential flatness (?)

4. PDE → FDE: Neutral system control (?)